Reply by Author to C. M. Hung and C. Fan

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HUNG and Fan's[1] improvement upon our analysis is a welcome one and significant whenever the detailed disturbance field away from the wall is of interest; indeed, it further strengthens our own earlier assertion[2] as to the importance of lateral pressure gradient effects in the study of small disturbances within highly nonuniform boundary-layer flows. We would like to re-emphasize, however, that our solution is exact near the wall (it is readily shown to be identical with Hung and Fan's theory as $y \to y_f$), gives acceptable agreement with experiment as Ref. 1 shows, and has proven a valuable ingredient in the theoretical analysis of other important problems such as heat and mass transfer disturbances,[3] ablation surface cross-hatching phenomena,[4] and liquid film stability in high-speed gas streams.[5]

Secondly, it is pointed out that the apparent disagreement in Fig. 4 of Ref. 1 regarding the phase angle variation is due entirely to a different choice of the reference angle: in our work, the local pressure phase was measured relative to the corresponding *local* maximum slope point pertaining to a wavy wall at this y, whereas Hung and Fan's pressure phasing is respect to the *fixed* location $y = y_f$. When this difference is accounted for, the phase angles given by the two analyses are the same; the only difference between them is the amplitude results for $y \gg y_f$.

Third, some remarks on the suitable choice of an analytical model of the Mach number (velocity) profile are deemed appropriate here. Owing to the presence of the $\partial M/\partial y$ term in the Lighthill pressure perturbation equation [their Eq. (1), our Eq. (2)], the velocity profile must be carefully chosen to yield a slope variation which is both continuous and nonsingular across the boundary layer while vanishing at its edge. The one-seventh power law profile used in the turbulent case by Hung and Fan does not satisfy the latter requirement and furthermore becomes inaccurate in precisely the turbulent law of the wall region where the effective wall conditions must be applied at $y = y_f$. Use of such a profile can give inaccurate solutions of the Lighthill equation as we found to be the case in our own earlier studies; consequently, we used instead a more realistic profile model[6] that obeys the proper law of the wall behavior for small y while giving a vanishing $\partial M/\partial y$ at $y = \delta$. This model is described by the following equations:

$$\frac{d(u/u_e)}{d\eta} \simeq \frac{c_f}{2} Re_\delta \frac{T}{T_e}\left(\frac{\tau/\tau_w}{\nu_{\text{eff}}/\nu_e}\right) \tag{1}$$

where

$$c_f \simeq 3.70 \frac{T_e}{T^*}\left\{\log_{10}\left[\left(\frac{T_e}{T^*}\right)^{1+\omega} Re_x\right]\right\}^{-2.58} \tag{2}$$

$$\frac{Re_\delta}{Re_x} = \frac{(n+2)(n+3)}{2n} c_f \left[1 + \frac{n(\gamma-1)M_e^2}{2(n+1)(n+2)} + \frac{T_w - T_e}{(n+1)T_e}\right] \tag{3}$$

$$\frac{T^*}{T_e} \simeq 1 + 0.044\, M_e^2 + 0.50\left(\frac{T_w}{T_e} - 1\right) \tag{4}$$

with T given by the Crocco energy equation integral, $\eta \equiv y/\delta$, $\tau/\tau_w \simeq 1 - 3\eta^2 + 2\eta^3$, and ν_{eff} is the piecewise-continuous effective eddy viscosity function shown here in Fig. 1. As illustrated in Fig. 2 of the present Note this model gave good agreement with experimental turbulent Mach number profiles.[2] The lack of agreement of Hung and Fan's Fig. 3 amplitude results with ours near the wall (where in fact they should be identical) is probably attributable to the inaccuracies associated with their profile choice.

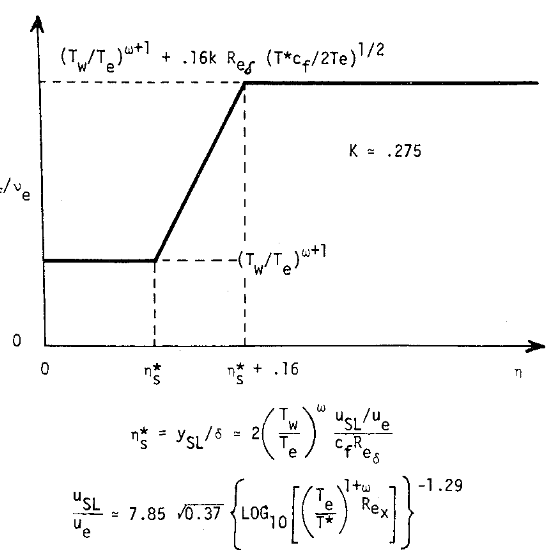

Fig. 1 Analytical model of turbulent boundary-layer profile.

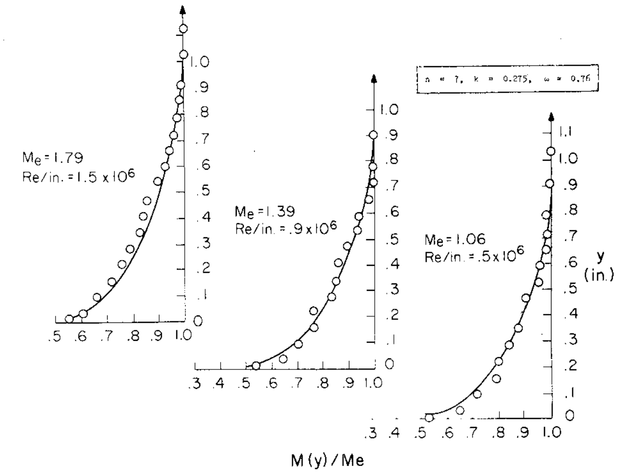

Fig. 2 Comparison of profile model with experiments of Inger and Williams.[2]

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[6] Williams, E. P. and Inger, G. R., "Final Report: Investigations of Ablation Surface Cross-Hatching," SAMSO TR-70-246, June 1970, McDonnell-Douglas Astronautics Co.—West, Los Angeles, Calif.